OCR
Oxford Cambridge and RSA

Wednesday 16 May 2018 – Morning

AS GCE MATHEMATICS (MEI)

4751/01 Introduction to Advanced Mathematics (C1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4751/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
None

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

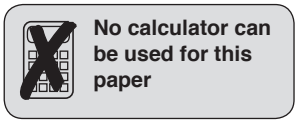
INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



Section A (36 marks)

1 Simplify $(5a^2c)^3 \times 2a^4c^{-5}$. **[2]**

2 Find the equation of the line joining the points $(-1, 9)$ and $(2, -3)$, giving your answer in the form $y = mx + c$. State the coordinates of the points where this line intersects the axes. **[5]**

3 Find the value of

(i) $\left(2\frac{1}{4}\right)^{-2}$, **[2]**

(ii) $(8000)^{\frac{2}{3}}$. **[2]**

4 For the following equation, express x in terms of y.

$$\frac{x}{3y} = \frac{2x+1}{y+2}$$ **[4]**

5 Find the coordinates of the point of intersection of the lines $y = 4x + 3$ and $3x + 2y = 9$. **[4]**

6 Find the term that is independent of x in the binomial expansion of $\left(\frac{1}{x} - 3x\right)^6$. **[3]**

7 **(i)** Express $\sqrt{28} + 3\sqrt{175}$ in the form $a\sqrt{b}$, where a and b are integers and b is as small as possible. **[2]**

(ii) Simplify $\frac{6}{5-\sqrt{2}} - \frac{3\sqrt{2}}{5+\sqrt{2}}$, giving your answer in the form $\frac{a+b\sqrt{2}}{c}$, where a, b and c are integers. **[3]**

8 For each of the following pairs of sentences A and B, give a reason why the statement A $\Leftrightarrow$ B is false and write either '$A \Rightarrow B$' or '$A \Leftarrow B$' to show the correct relationship.

(i) A: n is positive.
B: $n^2 + 6$ is positive. **[2]**

(ii) A: The diagonals of a quadrilateral bisect each other but not at right angles.
B: The quadrilateral is a rectangle but not a square. **[2]**

9 You are given that $f(x) = ax^3 + cx$ and that $f(-1) = 3$. You are also given that when $f(x)$ is divided by $(x - 4)$, the remainder is 108. Find the values of a and c. **[5]**

Section B (36 marks)

10 **(i)** Express $3x^2 - 9x + 5$ in the form $a(x + b)^2 + c$. Hence state the equation of the line of symmetry and the y-coordinate of the minimum point of the curve with equation $y = 3x^2 - 9x + 5$. **[6]**

(ii) Find the coordinates of the points where the graph of $y = 3x^2 - 9x + 5$ intersects the axes. Give your answers in an exact form. Hence state the solution of the inequality $3x^2 - 9x + 5 < 0$. **[4]**

11 You are given that $f(x) = (2x + 5)(x^2 - 5x + 4)$.

(i) Sketch the graph of $y = f(x)$. **[4]**

(ii) You are given that $g(x) = 2x^3 - 5x^2 - 17x + 48$. Show that $x = -3$ is a root of $g(x) = 0$ and that it is the only real root. **[6]**

(iii) Show that $y = g(x)$ is a translation of $y = f(x)$ by $\begin{pmatrix} 0 \\ k \end{pmatrix}$, finding the value of k. **[3]**

12

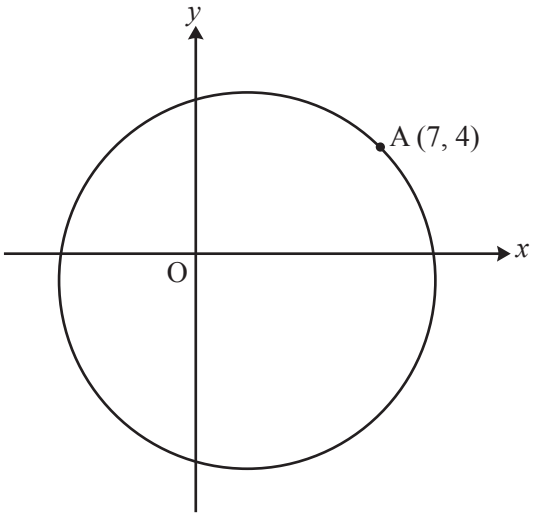


Fig. 12

Fig. 12 shows a sketch of the circle with equation $(x - 2)^2 + (y + 1)^2 = 50$. You are given that the point A (7, 4) lies on the circle.

(i) Write down the radius of this circle and the coordinates of its centre. **[2]**

(ii) The line L has equation $y = 2x - 10$ and passes through the point A (7, 4). Use algebra to find the coordinates of the point B where the line L meets the circle again. Hence show that the perpendicular distance from the centre of the circle to the line L is $\sqrt{5}$. **[6]**

(iii) Show that, when the line $y = 2x + k$ is a tangent to the circle, k satisfies the equation

$$k^2 + 10k - 225 = 0. \quad \textbf{[5]}$$

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$250a^{10}c^{-2}$ or $\frac{250a^{10}}{c^2}$	2 [2]	**B1** for 2 elements correct or **M1** for $\left(5a^2c\right)^3 = 125a^6c^3$	if p12, attached to this image, is blank, put BP on it. If it has an attempt at one or more parts, highlight the qn no on p12, then go to the qn and put a highlight by the side as a reminder that there is extra work and link p12 to this part
2			$y = -4x + 5$ www	3	**M1** for $m = \frac{9 - -3}{-1 - 2}\left[= -4\right]$ soi and **M1** for $y - 9 =$ *their m* $(x + 1)$ or $y + 3 =$ *their m* $(x - 2)$ or for correct ft elimination of fractions in eqn. of line joining two points	may be implicit in eqn eg $\frac{y-9}{-3-9} = \frac{x--1}{-1-2}$
			(0, 5) and (1.25, 0) oe, ft their eqn of line	2 [5]	**B1** for one correct; need not be in coordinate form; isw after acceptable ft ans found	as usual, no need to simplify fractions, but integers may not be left as fractions
3	(i)		$\frac{16}{81}$	2 [2]	**M1** for $\frac{81}{16}$ or for $\left(\frac{4}{9}\right)^2$ or for numerator or denominator correct	M0 for just converting mixed number to fraction or for $\left(\frac{1}{2.25}\right)^2$
3	(ii)		400	2 [2]	**M1** for $\sqrt[3]{8000} = 20$ soi or for $\sqrt[3]{64000000}$	

Question			Answer	Marks	Guidance	
4			$xy + 2x = 6xy + 3y$	M1	for correctly eliminating fractions and expanding brackets	
			$2x - 5xy = 3y$	M1	for correctly collecting x terms on one side and remaining terms on the other and simplifying	ft wrong first step
			$x(2 - 5y) = 3y$	M1	correctly factorising, ft their two or three term side with x terms	
			$[x =] \frac{3y}{2-5y}$ oe as final answer	M1	for correctly dividing by their factor – but **M0** if their factor is only one term since too simple in comparison	award full marks only if working fully correct
					SC for those who work to obtain $y =$: first correct step still gains **M1**; **M0** for collecting y terms on one side and remaining term on the other	ie if ft correct, those who work to obtain $y = \frac{2x}{5x+3}$ earn SC3
					then allow **M1** for correctly factorising, ft their two or three term side with y terms, then **M1** for correctly dividing by their factor to give final answer – must be simplified	
				[4]		

Question			Answer	Marks	Guidance	
5			$3x + 2(4x + 3) = 9$	M1	for subst to eliminate one variable; condone one error; or for multn or divn of one or both eqns to get a pair of coeffts the same, condoning one error	each M1 is for a correct, constructive step
			$11x = 3$	M1	for collecting terms and simplifying; condoning one error ft or for appropriate addn or subtn to eliminate a variable, condoning an error in one term;	for this M mark, ft for equiv difficulty
			(3/11, 45/11 oe)	A2 **[4]**	or $x = 3/11$, $y = 45/11$ oe isw allow **A1** for each coordinate	
6			selecting the term in $\left(\frac{1}{x}\right)^3(-3x)^3$	M1	condone wrong or omitted brackets; may be implied by $(-3)^3$ or by −27 selected	
			${}^6C_3 = 20$ soi	M1	may be part of a fully correct row in Pascal's triangle	mark to advantage of candidate if choice eg if correct row seen but wrong element selected
			−540	A1 **[3]**	allow **B2** for −540 included as part of expansion but not identified as required term – ignore errors in other terms allow **B3** for −540 obtained and identified by multiplying out brackets, ignoring errors in other terms, otherwise M0 for this method	
7	(i)		$17\sqrt{7}$	2 **[2]**	**M1** for $\sqrt{28} = 2\sqrt{7}$ soi or for $\sqrt{175} = 5\sqrt{7}$	

Question			Answer	Marks	Guidance	
7	(ii)		$\frac{36-9\sqrt{2}}{23}$ or $\frac{36+-9\sqrt{2}}{23}$ or $a = 36$, $b = -9$, $c = 23$	3 [3]	**B1** for denominator; **B2** for numerator or **M1** for three terms correct in $30+6\sqrt{2}-15\sqrt{2}+6$	
8	(i)		when n is negative, n^2 is positive and so $n^2 + 6$ is positive	B1	oe with a valid number, or equivalent explanation eg may use $n = 0$	not sufft to say n doesn't have to be positive for $n^2 + 6$ to be positive; but allow B1 for statement such as '$n^2 \geq 0$ whatever n is, so $n^2 + 6$ is always positive' B0 for 'n^2 is always positive so n can be negative'
			$A \Rightarrow B$	B1 [2]	condone $B \Leftarrow A$ **B0** if no attempt at explanation (explanation does not need to gain a mark)	Do not accept $A \rightarrow B$ oe or just $\Rightarrow$ or $\Leftarrow$ with no A and B
	(ii)		the diagonals of a parallelogram also bisect each other, not at 90°	B1	oe for other valid statement/sketch eg 'A is true for parallelograms as well' but **B0** for eg 'parallelograms also have diagonals meeting not at 90°' – need diagonals bisecting each other as well; **B0** if eg square or rhombus or kite or trapezium etc also included as having diagonals bisecting each other, not at 90°	reference merely to 'other shapes' having diagonals bisecting each other but not at 90° is not sufficient; if explanation has words, ignore sketches unless referred to in words; if explanation is by sketches only, they must have diagonals drawn, approx. bisecting each other not at right angles but need not be ruled
			$A \Leftarrow B$	B1 [2]	condone $B \Rightarrow A$ **B0** if no attempt at explanation (condone explanation of why the symbol they give is true or any sketch) - (explanation does not need to gain a mark)	Do not accept $A \leftarrow B$ oe or just $\Rightarrow$ or $\Leftarrow$ with no A and B

Question			Answer	Marks	Guidance	
9			$-a - c = 3$	B1	accept $(-1)^3a$ instead of $-a$	
			$64a + 4c = 108$	B1	accept $(4)^3a$ instead of $64a$	may also be obtained after long division etc
			Correct method for eliminating one variable, condoning one further error	M1	dep on two equations in a and c and at least B1 earned	
			$a = 2$, $c = -5$	A2	**A1** for one correct if M0 but a and c both correct, allow **SC1**	
				[5]		
10	**(i)**		$3(x - 1.5)^2 - 1.75$ oe in fractions, www	B4	**B1** for each of $a = 3$, $b = -1.5$ and **B2** for $c = -1.75$ or **M1** for $5 - 3 \times 1.5^2$ or ft soi or for $5/3 - 1.5^2$ or ft soi	ignore '= 0' if brackets are there, condone missing power of 2
			Line of symmetry is $x =$ – their b	B1	must ft; if correct, $x = 1.5$	for last two B marks, do not allow those starting again since not 'hence'
			min y = their c	B1	must ft; if correct, $y = -1.75$ **B0** for just min pt = (1.5, – 1.75) oe statement needed not just sketch with – 1.75 marked on y-axis	
				[6]		

Question			Answer	Marks	Guidance	
10	(ii)		intersects y-axis at (0, 5)	B1		
			intersects x-axis at $\left(\frac{9\pm\sqrt{21}}{6},0\right)$ or $\left(\frac{3}{2}\pm\sqrt{\frac{7}{12}},0\right)$ or $x=\frac{9\pm\sqrt{21}}{6}$ or $x=\frac{3}{2}\pm\sqrt{\frac{7}{12}}$	B2	**M1** for quadratic formula used or ft from their completing the square in (i), with at most one error; condone answers not in coordinate form; isw after correct x values obtained	
			$\frac{9-\sqrt{21}}{6}<x<\frac{9+\sqrt{21}}{6}$ or ft	B1 **[4]**	ft only for soln using surds; allow ft from wrongly simplified surds; do not accept two separate inequalities	
11	(i)		graph of cubic correct way up	B1	**B0** if stops at x-axis	must not have any ruled sections; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; allow max on y-axis or in 1st or 2nd quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)
			crossing x-axis at −5/2, 1 and 4	B2	on graph or nearby; may be in coordinate form; **M1** for $x^2 - 5x + 4 = (x - 4)(x - 1)$ or for roots 4 and 1 found mark intent for intersections with both axes	allow if no graph, but marked on x-axis condone intercepts for x and / or y given as reversed coordinates
			crossing y-axis at 20	B1 **[4]**	or $x = 0$, $y = 20$ seen if consistent with graph drawn	allow if no graph, but eg B0 for graph with intn on y-axis nowhere near their indicated 20

Question			Answer	Marks	Guidance	
11	(ii)		$g(-3) = 2 \times (-3)^3 - 5 \times 9 - 17 \times -3 + 48$ $= -54 - 45 + 51 + 48$ $= 0$	B1	condone $(-3)^3$ instead of -27 etc, but next step of working must be shown correctly or **B1** for correct division of $g(x)$ by $(x + 3)$ with remainder 0 and the conclusion immediately following this (or explicitly connected to it) that $g(-3) = 0$ or that -3 is a root of $g(x)$ oe	B0 for just $x + 3$ is a factor or for $x + 3$ is a root
			$(x + 3)$ used or stated as factor	M1		
			correctly finding other factor as $2x^2 - 11x + 16$	B2	accept $b = -11$ found **M1** for correct division of cubic by $(x + 3)$ as far as obtaining $2x^2 - 11x$ (may be in grid) or for two correct terms of $2x^2 - 11x + 16$ obtained by inspection	
			$121 - 128$ isw or -7	A1	for correct substitution into $b^2 - 4ac$ and obtaining negative (may be seen in formula); no ft from wrong factor	must be correctly simplified to at least the $121 - 128$ stage
			conclusion no real roots from quadratic factor/equation, so -3 is only real root of $g(x)$	A1 **[6]**	dep on previous A1; must refer back to original request, just 'no real roots' is not sufft they need to mention -3 or say 'so just one real root' or 'no more real roots'	
11	(iii)		$[f(x) =] 2x^3 - 5x^2 - 17x + 20$ with correct working	B2	**B1** if no working or **M1** for correct working condone inclusion of $+ k$ even if labelled as $f(x)$ instead of $g(x)$	if no working in (iii), check whether the relevant work has already been done in (i). If it has, tick it on the copy in the image zone and allow the mark, but only if $f(x)$ appears/is used in (iii).

Question			Answer	Marks	Guidance	
			$k = 28$ or g(x) is translation of f(x) by $\begin{pmatrix} 0 \\ 28 \end{pmatrix}$	B1 **[3]**	**B0** for just g(x) = f(x) + 28	B1 for $k = 28$ even if stated after no /wrong f(x) obtained
12	**(i)**		radius $\sqrt{50}$ isw wrong conversion to $5\sqrt{2}$ centre $(2, -1)$	B1 B1 **[2]**	**B1** for $5\sqrt{2}$	
12	**(ii)**					NB examiners must use annotation in this part; a tick where each mark is earned is sufficient
			$(x-2)^2 + (2x-9)^2 = 50$	M1	for subst from line into circle eqn; condone one error	eg condone omission of '=50' or having -11 instead of -9
			$5x^2 - 40x + 35$ [= 0]	M1	for simplifying to solvable form; condone one further error	
			$x = 7$ or 1	A1	condone omission of 7 and just using 1	
			B = $(1, -8)$	B1		
			midpt of AB = $\left(\frac{7+their1}{2}, \frac{4+their-8}{2}\right)$ or $(4, -2)$	M1	or length of AB found ft ($\sqrt{180}$ if correct) and Pythagoras used with ½ AB and r	Must use the coordinates of B since 'hence': so M0 for eqn of line through centre perp to AB and intersection with AB used to find mid point of AB or M0 for equation of AB and formula for dist of pt from line used
			distance = $\sqrt{5}$ correctly obtained (answer given)	A1 **[6]**		

Question			Answer	Marks	Guidance	
12	(iii)		$(x-2)^2 + (2x+k+1)^2 = 50$	M1	condone one error, eg omission of +1, but k must be included	
			$5x^2 + 4kx + k^2 + 2k - 45$ [= 0]	M1	condone one error; accept constant term $(k+1)^2 - 46$; must be rearranged to '=0' stage unless they go on to complete the square **M0** if wrong eqn used – no ft from original error, only condone one error from working with correct eqn	eg allow M1 for $5x^2 + 4kx + k^2 - 45$ [= 0]
			$b^2 - 4ac = 0$ oe soi	M1	may be earned near end allow for this condition quoted, even if then applied to wrong equation. It is sometimes earned at beginning	0 for just 'discriminant = 0' unless implied by later work
			$(4k)^2 - 4 \times 5 \times (k^2 + 2k - 45)$	M1	for correct substitution ft into $b^2 - 4ac$, dep on first M1 earned; brackets / signs must be correct	can be earned in formula (ignore rest of formula)
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A1 **[5]**	NB mark working not answer	

Question			Answer	Marks	Guidance	
12	(iii)		**method 2**	**or**		
			line perp to $y = 2x + k$ through centre is $y = -\frac{1}{2}x$ oe	M1	condone attempt $y = -\frac{1}{2}x + n$, with $n \neq 0$	M0 for just $y = -\frac{1}{2}x + c$ with no attempt to subst $(2, -1)$ to find c
			finding intersection with $y = 2x + k$ [if correct, $x = -\frac{2}{5}k, y = \frac{1}{5}k$]	M1	allow for finding intn of $y = 2x + k$ and line with grad – ½ but error in constant	
			$\left(2 + \frac{2}{5}k\right)^2 + \left(-1 - \frac{1}{5}k\right)^2 = 50$ oe	M1	for correct substitution ft into circle equation, dep on first M1 earned; brackets / signs must be correct	using distance from centre = radius, or point of intersection being on circle
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A2	NB mark working not answer; **A1** for correct expansion of brackets or correctly eliminating fractions as first step, working with correct equation only	A1 for $4 + \frac{8k}{5} + \frac{4k^2}{25} + 1 + \frac{2k}{5} + \frac{k^2}{25} = 50$ oe or $(10 + 2k)^2 + (5 + k)^2 = 1250$ oe
				[5]		

Question			Answer	Marks	Guidance	
12	(iii)		**method 3**	**or**		
			line perp to $y = 2x + k$ through centre is $y = -\frac{1}{2}x$ oe	M1	condone attempt $y = -\frac{1}{2}x + n$, with $n \neq 0$	M0 for just $y = -\frac{1}{2}x + c$ with no attempt to subst $(2, -1)$ to find c
			finding intn of their perp line with circle: $(x - 2)^2 + (-\frac{1}{2}x + 1)^2 = 50$ and simplifying to solvable form	M1	allow using line with grad $-\frac{1}{2}$ but error in constant	
			$x = \frac{4 \pm \sqrt{160}}{2}, y = \frac{-4 \text{ m} \sqrt{160}}{4}$ oe	A1		
			using $y = 2x + k$ to obtain k $k = -5 \pm 5\sqrt{10}$ oe if correct	M1	dep on previous Ms; or may use eqn of line gradient 2 through each of these points and compare results with $y = 2x + k$	
			$(k + 5)^2 = 250$ and correct working to obtain given answer $k^2 + 10k - 225 = 0$.	A1	NB mark working not answer; or allow subst of $k = -5 \pm 5\sqrt{10}$ oe into $k^2 + 10k - 225 = 0$ and showing consistent	
				[5]		

Question			Answer	Marks	Guidance	
12	(iii)		**method 4**	**or**		
			using calculus: $2x-4+2y\frac{dy}{dx}+\frac{dy}{dx}=0$ and subst $\frac{dy}{dx}=2$ [if correct, $2x-4+4y+2=0$]	M1	condone one error	
			using $y=2x+k$, subst and solving: [if correct, $2x-4+4(2k+1)+2=0$ and $x=-\frac{2k}{5}, y=\frac{k}{5}$]	M1	condone one error	[cf method 2: more work to be done by method 4 to get to the stage of finding the point of contact in terms of k]
			$\left(2+\frac{2}{5}k\right)^2+\left(-1-\frac{1}{5}k\right)^2=50$ oe	M1	for correct substitution ft into circle equation, dep on first M1 earned; brackets / signs must be correct	using distance from centre = radius, or point of contact being on circle
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A2	NB mark working not answer; **A1** for correct expansion of brackets or correctly eliminating fractions as first step, working with correct equation only	A1 for $4+\frac{8k}{5}+\frac{4k^2}{25}+1+\frac{2k}{5}+\frac{k^2}{25}=50$ oe or $(10+2k)^2+(5+k)^2=1250$ oe
				[5]		